AQA Computer Science A-Level

4.4.5 A model of computation

Past Paper Questions

June 2011 Comp 3

11 A particular Turing machine has states S_1, S_2, S_3 and S_4. S_1 is the start state and S_4 is the stop state. The machine uses one tape which is infinitely long in one direction to store data. The machine's alphabet is 1, □. The symbol □ is used to indicate a blank cell on the tape.

The transition rules for this Turing machine can be expressed as a transition function δ. Rules are written in the form:

$$\delta(\text{Current State, Input Symbol}) = (\text{Next State, Output Symbol, Movement})$$

So, for example, the rule:

$$\delta(S_1, 1) = (S_1, 1, \rightarrow)$$

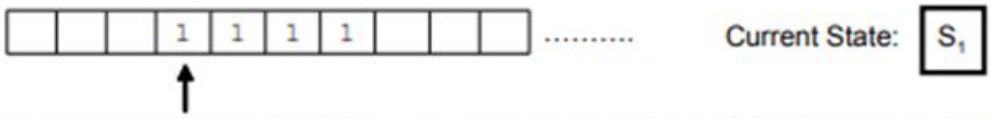
means:

IF the machine is currently in state S_1 AND the input symbol read from the tape is 1
THEN the machine should remain in state S_1, write a 1 to the tape and move the read/write head one cell to the right

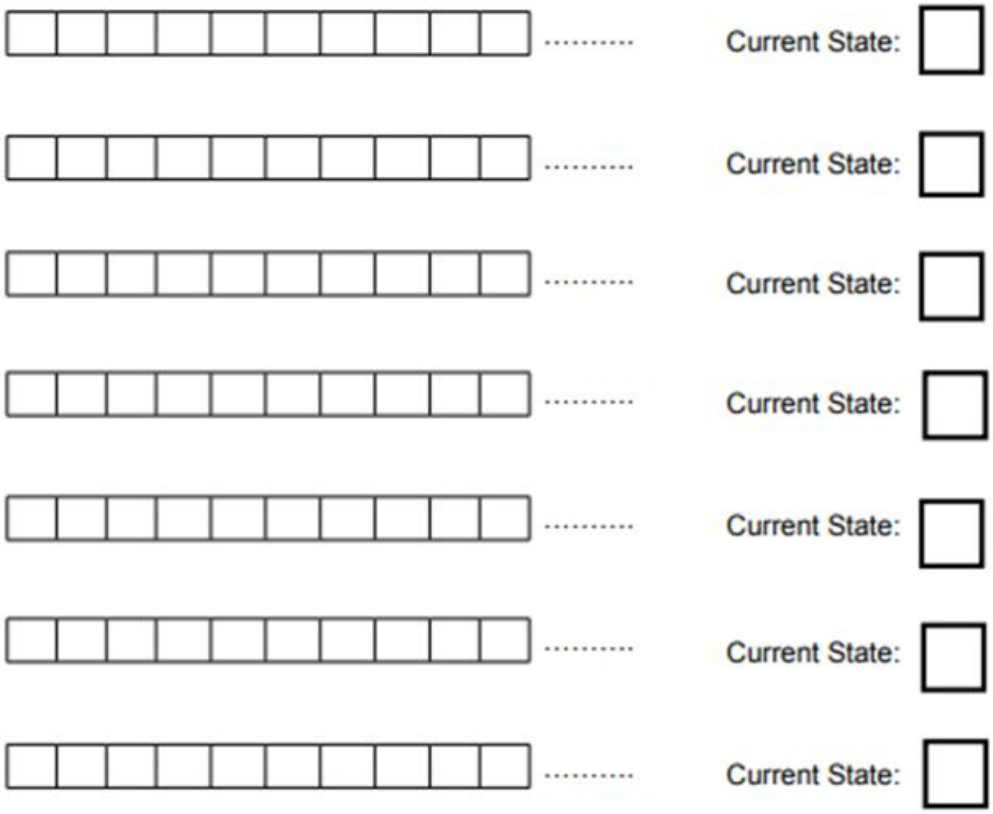
The machine's transition function, δ, is defined by:

$$\begin{aligned}
\delta(S_1, 1) &= (S_1, 1, \rightarrow) \\
\delta(S_1, \square) &= (S_2, \square, \leftarrow) \\
\delta(S_2, 1) &= (S_3, \square, \leftarrow) \\
\delta(S_3, 1) &= (S_4, \square, \leftarrow)
\end{aligned}$$

11 (a) The Turing machine is carrying out a computation. The machine starts in state S_1 with the string `1111` on the tape. All other cells contain the blank symbol, □. The read/write head is positioned at the leftmost `1`, as indicated by the arrow.



Trace the computation of the Turing machine, using the transition function δ. Show the contents of the tape, the current position of the read/write head and the current state as the input symbols are processed.



(6 marks)

11 (b) Explain what this Turing machine does.

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(1 mark)

11 (c) Explain what a *Universal Turing machine* is.

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(2 marks)

June 2013 Comp 3

7 A particular Turing machine has states S_B, S_0, S_1, S_R and S_T. S_B is the start state and S_T is the stop state. The machine stores data on a single tape which is infinitely long in one direction. The machine's alphabet is 0, 1, #, x, y and □ where □ is the symbol used to indicate a blank cell on the tape.

The transition rules for this Turing machine can be expressed as a transition function δ. Rules are written in the form:

$$\delta \text{ (Current State, Input Symbol)} = \text{(Next State, Output Symbol, Movement)}$$

So, for example, the rule:

$$\delta (S_B, 1) = (S_1, y, \rightarrow)$$

means:

IF the machine is currently in state S_B AND the input symbol read from the tape is 1

THEN the machine should change to state S_1, write a y to the tape and move the read/write head one cell to the right

The machine's transition function, δ, is defined by:

$$\begin{aligned}
\delta (S_B, 0) &= (S_0, x, \rightarrow) \\
\delta (S_B, 1) &= (S_1, y, \rightarrow) \\
\delta (S_B, \#) &= (S_T, \#, \rightarrow)
\end{aligned}$$

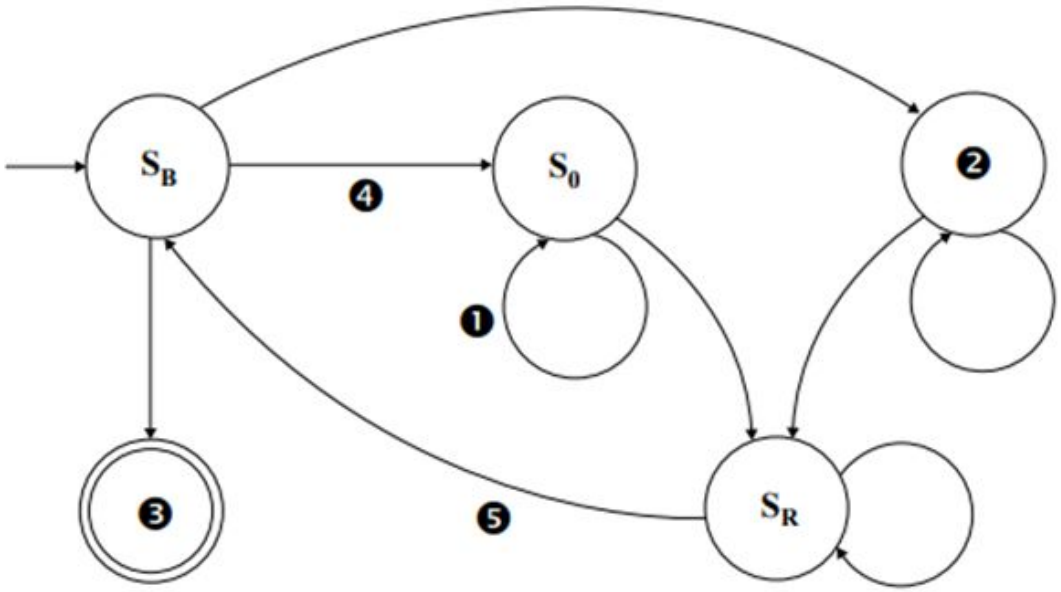
$$\begin{aligned}
\delta (S_0, 0) &= (S_0, 0, \rightarrow) \\
\delta (S_0, 1) &= (S_0, 1, \rightarrow) \\
\delta (S_0, \#) &= (S_0, \#, \rightarrow) \\
\delta (S_0, \square) &= (S_R, 0, \leftarrow)
\end{aligned}$$

$$\begin{aligned}
\delta (S_1, 0) &= (S_1, 0, \rightarrow) \\
\delta (S_1, 1) &= (S_1, 1, \rightarrow) \\
\delta (S_1, \#) &= (S_1, \#, \rightarrow) \\
\delta (S_1, \square) &= (S_R, 1, \leftarrow)
\end{aligned}$$

$$\begin{aligned}
\delta (S_R, 0) &= (S_R, 0, \leftarrow) \\
\delta (S_R, 1) &= (S_R, 1, \leftarrow) \\
\delta (S_R, \#) &= (S_R, \#, \leftarrow) \\
\delta (S_R, x) &= (S_B, 0, \rightarrow) \\
\delta (S_R, y) &= (S_B, 1, \rightarrow)
\end{aligned}$$

Figure 9 shows an unlabelled finite state transition diagram for this machine. Some of the state transition arrows represent more than one of the machine's transition rules. For example, the arrow labeled ❶ represents the three rules: $\delta (S_0, 0) = (S_0, 0, \rightarrow)$, $\delta (S_0, 1) = (S_0, 1, \rightarrow)$ and $\delta (S_0, \#) = (S_0, \#, \rightarrow)$.

Figure 9



7 (a) (i) Which states are represented by the labels ❷ and ❸ in **Figure 9**?

❷ ..... ❸ .....

(1 mark)

7 (a) (ii) Which of the machine's transition rule(s) is/are represented by the arrow labelled ❹ in **Figure 9**?

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(1 mark)

7 (a) (iii) Which of the machine's transition rule(s) is/are represented by the arrow labelled ❺ in **Figure 9**?

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(1 mark)

The machine's transition rule, δ, is repeated here so that you can answer question **7(b)** without having to turn back in the question paper booklet.

$$\delta(S_B, 0) = (S_0, x, \rightarrow)$$
$$\delta(S_B, 1) = (S_1, y, \rightarrow)$$
$$\delta(S_B, \#) = (S_T, \#, \rightarrow)$$

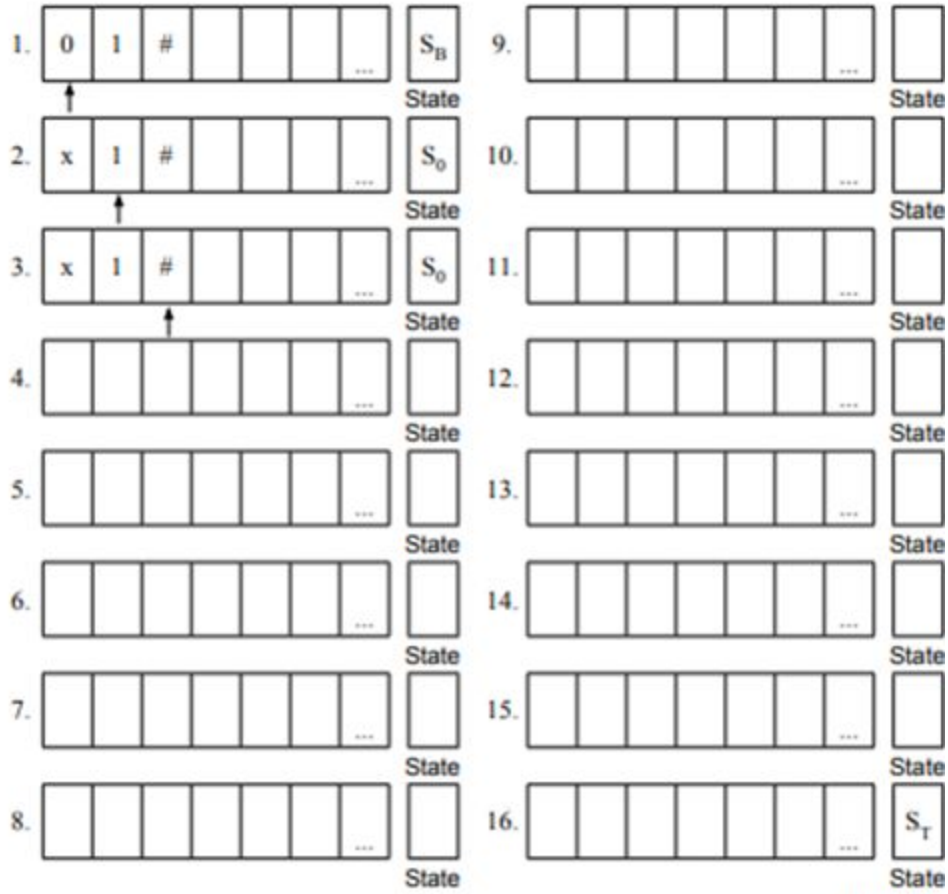
$$\delta(S_0, 0) = (S_0, 0, \rightarrow)$$
$$\delta(S_0, 1) = (S_0, 1, \rightarrow)$$
$$\delta(S_0, \#) = (S_0, \#, \rightarrow)$$
$$\delta(S_0, \square) = (S_R, 0, \leftarrow)$$

$$\delta(S_1, 0) = (S_1, 0, \rightarrow)$$
$$\delta(S_1, 1) = (S_1, 1, \rightarrow)$$
$$\delta(S_1, \#) = (S_1, \#, \rightarrow)$$
$$\delta(S_1, \square) = (S_R, 1, \leftarrow)$$

$$\delta(S_R, 0) = (S_R, 0, \leftarrow)$$
$$\delta(S_R, 1) = (S_R, 1, \leftarrow)$$
$$\delta(S_R, \#) = (S_R, \#, \leftarrow)$$
$$\delta(S_R, x) = (S_B, 0, \rightarrow)$$
$$\delta(S_R, y) = (S_B, 1, \rightarrow)$$

7 (b) This Turing machine is carrying out a computation. The machine starts in state S_B with the string $01\#$ on the tape. All other cells contain the blank symbol, $\square$ (not shown).

Trace the computation of the Turing machine, using the transition function δ. Show the contents of the tape, the current position of the read/write head and the current state as the input symbols are processed. The first three steps and final state have been completed for you.



(6 marks)

7 (c) (i) Describe the purpose of the symbols x and y in this Turing machine's alphabet.

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(1 mark)

7 (c) (ii) What does the Turing machine do?

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(1 mark)